Modal analysis of surface wave-fluid-structure interaction problem

V. V. Mokeyev

Cheliabinsk State Technical University

A technique for dynamic analysis of the interaction of a structure with a linear fluid is presented. It is based on finite element discretization and determination of response by modal superposition analysis. The eigenvalue problem is solved with the help of frequency condensation method. It is shown that this method is an efficient means for large fluid-structure eigenvalue problem. Numerical example is given to demonstrate the good results of frequency condensation method.

Introduction

The solution of fluid-structure interaction problem is considered. For fluid the following assumptions hold:

1. The velocity in the fluid is small so that convective terms can be neglected.
2. The shear tension in the fluid is proportional velocity.
3. The pressure alone describes the state of the fluid.
4. The viscosity of fluid is neglected.

The finite element formulation of problem solution is used. Therefore the behaviour of fluid is described in mesh of finite elements with the pressures as nodal variables, behaviour of free surface is described in mesh of finite elements with displacements normal to the static free surface as nodal variables, and behaviour of structure is described in mesh of finite elements with displacements as nodal ones. Such finite element formulation results in matrix equation in form

$$\mathbf{K}\delta + \mathbf{C}\dot{\delta} + \mathbf{M}\ddot{\delta} = \mathbf{F}\,, \tag{1}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{M_p} & \mathbf{L}^T & \mathbf{0} \\ \mathbf{L} & -\mathbf{H} & \mathbf{Q}^T \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K_p} + \mathbf{K}^* & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \rho^{-1}\mathbf{Q} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C_p} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \delta = \begin{Bmatrix} \mathbf{u} \\ \gamma \\ \mathbf{p} \end{Bmatrix}, \quad \mathbf{F} = \begin{Bmatrix} \mathbf{F}_p \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}.$$

Here $\mathbf{u}$ is vector of nodal displacement of structure and normal displacement of free surface of fluid, γ is vector of nodal displacement potential of fluid, $\mathbf{p}$ is vector of nodal pressure of fluid, $\mathbf{K}_p$,$\mathbf{C}_p$,$\mathbf{M}_p$ are structural stiffness, viscosity and mass matrices, $\mathbf{L}$ is coupling matrix, $\mathbf{H}$,$\mathbf{Q}$ are volumetric fluid matrix and fluid inertial mass matrix, $\mathbf{K}^*$ is stiffness matrix due to the free surface, $\mathbf{F}_p$ is vector of external forces, acting on elastic structure, ρ is density of fluid. The expressions for matrices of finite elements, from which the matrices $\mathbf{K}_p$,$\mathbf{C}_p$,$\mathbf{M}_p$, $\mathbf{H}$, $\mathbf{Q}$, $\mathbf{L}$, $\mathbf{K}^*$ are assembled, can be found in References 1and 2. In the case of an incompressible fluid, matrix $\mathbf{Q}$ is equal to zero.

Solution method

The equation (1) can be solved using normal modes as the generalised co-ordinates by making the transformation $\delta = \Delta\xi(t)$ in which $\xi(t)$ is modal displacement vector, Δ is eigenvectors matrix corresponding to eigenvalues w^2 satisfying the eigenvalue problem

$$\left(\mathbf{K} - w^2\mathbf{M}\right)\delta_0 = \mathbf{0} \ . \tag{2}$$

The eigenvalue problem (2) is solved with the help of frequency condensation method [3]. This method is based on reduction of matrices size. As distinct from the Guyan method [2] the frequency condensation method is based on approximation preserving eigenvalues in a preset frequency interval and effective procedure of masters and slaves selection. The frequency condensation algorithm is described in more details in References 3 and 4. Features of this method application to surface wave-fluid-structure interaction problem are considered below.

(a) The matrices condensation step involves the reduction of the matrices size by the eliminating of vector δ_0 components. The components of vector γ_0 are eliminated by Gaussian process, and components of vectors $\mathbf{u}_0$ and $\mathbf{p}_0$ are eliminated by the frequency condensation process. The scaling for alignment of the elements of matrices $\mathbf{K}_p$, $\mathbf{M}_p$, $\mathbf{Q}$ is implemented on condensation step, that provides the matrix condensation process stability.

(b) In order to minimise condensation error it is necessary to eliminate the components of vector γ_0 at first. However, it results to increase of working matrices size and according to necessary operative memory for the eigenvalue problem solution. Therefore, if the components of vectors $\mathbf{u}_0$, $\mathbf{p}_0$, γ_0 are eliminated simultaneously, it is necessary to be limited the errors introduced in equations of the components of vector γ_0. For this purpose it is necessary to check the fulfilment of next condition

$$\left|\frac{e_{ii}^{s}\left(w_{md}\right)}{k_{ii}}\right| \le u$$

in addition to verification of conditions described in Reference 3. Here e_{ii}^{s} is error, introduced in equation of variable δ_{0i}, when variable δ_{0s} is eliminated [3]; k_{ii} is diagonal element of matrix $\mathbf{K}$ on current condensation step; $w_{md}^2 = \left(w_1^2 + w_2^2\right)/2$; w_1^2 and w_2^2 are boundary frequencies of preset frequency interval. If the variable δ_{0i} is component of vector γ_0, than the value u is equal to 0.0004. If the variable δ_{0i} is component of vector $\mathbf{u}_0$ or $\mathbf{p}_0$, than the value u is equal to 0.02.

(c) Since the matrix $\mathbf{H}$ is singular, the elimination of the last component of vector γ_0 results "to division on zero". I shall explain it in more detail. For this purpose the matrices $\mathbf{H}$, $\mathbf{L}$, $\mathbf{Q}$ and vector γ_0 are presented in the form

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_m & \mathbf{H}_1^T \\ \mathbf{H}_1 & h_{m+1} \end{bmatrix} \ , \quad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_m & \mathbf{Q}_1^T \\ \mathbf{Q}_1 & q_{m+1} \end{bmatrix} \ , \quad \mathbf{L} = \begin{bmatrix} \mathbf{L}_m & \mathbf{L}_1 \end{bmatrix} \ ,$$

$$\gamma_0 = \begin{Bmatrix} \gamma_{0m} \\ \gamma_{0m+1} \end{Bmatrix} \ , \quad \mathbf{p}_0 = \begin{Bmatrix} \mathbf{p}_{0m} \\ \mathbf{p}_{0m+1} \end{Bmatrix} \ ,$$

where $m+1$ is dimension of vector γ_0, n is dimension of vector $\mathbf{u}_0$, $\mathbf{H}_m$, $\mathbf{Q}_m$ are matrices of dimension $m \cdot m$, $\mathbf{H}_1$, $\mathbf{Q}_1$ are matrices of dimension $1 \times m$, h_{m+1}, q_{m+1} are diagonal elements of matrices $\mathbf{H}$ and $\mathbf{Q}$, $\mathbf{L}_m$ is matrix of dimension $n \cdot m$, $\mathbf{L}_1$ is matrix of dimension $1 \times n$.

Since the matrix $\mathbf{H}_m$ is non-singular, the components of vector γ_{0m} can be eliminated on Gaussian process. After such elimination the equation (2) will have form

$$\left(-\omega^2 \begin{bmatrix} \mathbf{M}_p + \mathbf{M}_a & \mathbf{L}_1^T + \mathbf{L}_{a1}^T & \mathbf{S}_{am}^T & \mathbf{S}_{a1}^T \\ \mathbf{L}_1 + \mathbf{L}_{a1} & -(h_{m+1} + h_a) & \mathbf{Q}_1^T + \mathbf{Q}_{a1}^T & q_{m+1} + q_a \\ \mathbf{S}_{am} & \mathbf{Q}_1 + \mathbf{Q}_{a1} & \mathbf{G}_{am} & \mathbf{G}_{a1}^T \\ \mathbf{S}_{a1} & q_{m+1} + q_a & \mathbf{G}_{a1} & g_a \end{bmatrix} + \right. \tag{3}$$

$$\left. + \begin{bmatrix} \mathbf{K}_p + \mathbf{K}^* & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \rho^{-1}\mathbf{Q}_m & \rho^{-1}\mathbf{Q}_1^T \\ \mathbf{0} & \mathbf{0} & \rho^{-1}\mathbf{Q}_1 & \rho^{-1} q_{m+1} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{u}_0 \\ \boldsymbol{\gamma}_{0m+1} \\ \mathbf{p}_{0m} \\ \mathbf{p}_{0m+1} \end{Bmatrix} = \mathbf{0},$$

where

$$\mathbf{M}_a = \mathbf{L}_m^T \mathbf{H}_m^{-1} \mathbf{L}_m \ , \quad \mathbf{L}_{a1} = \mathbf{H}_1^T \mathbf{H}_m^{-1} \mathbf{L}_m \ , \quad \mathbf{S}_{am} = \mathbf{Q}_m^T \mathbf{H}_m^{-1} \mathbf{L}_m$$

$$\mathbf{S}_{a1} = \mathbf{Q}_1^T \mathbf{H}_m^{-1} \mathbf{L}_m \ , \quad \mathbf{Q}_{a1} = -\mathbf{H}_1^T \mathbf{H}_m^{-1} \mathbf{Q}_m \ , \quad q_{a1} = -\mathbf{H}_1^T \mathbf{H}_m^{-1} \mathbf{Q}_1$$

$$\mathbf{G}_{a1} = \mathbf{Q}_m^T \mathbf{H}_m^{-1} \mathbf{Q}_1 \ , \quad \mathbf{G}_{am} = \mathbf{Q}_m^T \mathbf{H}_m^{-1} \mathbf{Q}_m, \quad h_a = -\mathbf{H}_1^T \mathbf{H}_m^{-1} \mathbf{H}_1 ,$$

$$g_a = \mathbf{Q}_1^T \mathbf{H}_m^{-1} \mathbf{Q}_1 .$$

The matrix element $\mathbf{h}_{m+1}+\mathbf{h}_a$ is equal to zero, as the matrix $\mathbf{H}$ is singular. Therefore, the elimination of γ_{0m+1} is impossible on standard Gaussian process. The elimination of γ_{0m+1} is possible by two ways. (1) In first way value $\alpha\mathbf{h}_{m+1}$ is added to "zero" and then γ_{0m+1} is eliminated on Gaussian process. The actual value α depends naturally on the number of significant digits used in the computation. If the computation is implemented with double precision words than α is equal to $10^{-6}...10^{-8}$.

(2) In second way the component γ_{0m+1} is eliminated with the help of expression $\boldsymbol{\gamma} = (\omega^2\rho)^{-1}\mathbf{p}$. Thus after the component γ_{0m+1} elimination the equation (3) has form

$$\left(\begin{bmatrix} \mathbf{K}_p & \mathbf{0} & -\rho^{-1}(\mathbf{L}_1 + \mathbf{L}_{a1})^T \\ \mathbf{0} & \rho^{-1}\mathbf{Q}_m & -\rho^{-1}\mathbf{Q}_{a1}^T \\ -\rho^{-1}(\mathbf{L}_1 + \mathbf{L}_{a1}) & -\rho^{-1}\mathbf{Q}_{a1} & -\rho^{-1}(q_{m+1} + 2q_a) \end{bmatrix} - \right.$$

$$\left. -\omega^2 \begin{bmatrix} \mathbf{M}_p + \mathbf{M}_a & \mathbf{S}_{am}^T & \mathbf{S}_{a1}^T \\ \mathbf{S}_{am} & \mathbf{G}_{am} & \mathbf{G}_{a1}^T \\ \mathbf{S}_{a1} & \mathbf{G}_{a1} & g_a \end{bmatrix} \right) \begin{Bmatrix} u_0 \\ p_{0m} \\ p_{0m+1} \end{Bmatrix} = \mathbf{0}.$$

Note that if the first way is used than the mass matrix elements are modified, and if the second way is used than the stiffness matrix elements are modified.

The accuracy of given ways is demonstrated on example of free axisymmetric vibration of cylindrical shell filled by gasoline. The bottom of shell is flat and absolutely rigid. The bottom edge of cylindrical shell is fixed supported. The shell properties: radius is 1 m, thickness is 0.001 m, Young's modulus is 70 GPa, Poisson's ratio is 0.3, density is 2700 kg/m^3. The gasoline properties: velocity of sound is 1450 m/s^2, density is 800 kg/m^3.

As criterion of quality of calculated frequencies and shapes parameter c is used

$$c = \frac{\left\| \left(\mathbf{K} - w^2 \mathbf{M} \right) \delta_0 \right\|}{\left\| \mathbf{K} \delta_0 \right\|}.$$

The parameter c_i presents the ratio of residual's norm of equation (2) to elastic forces norm and characterizes the degree of approach w^2 and δ_0 to the corresponding exact values.

In Table 1 are compared natural frequencies and quality criterion obtained with help of first and second ways. In this Table the numbers of masters are presented. The results shows, that notwithstanding good agreement of natural frequencies, the calculation accuracy of the second way is much higher and number of masters is much lower.

Numerical example

To prove correct above described technique and its accuracy for the surface wave-fluid-structure system solution the free axisymmetric vibration hemispherical shell with water are considered. The shell top edge is clamped. The shell properties: radius is 5.08 m, thickness is 0.00254 m, Young's modulus is 10 GPa, Poisson's ratio is 0.3, density is 2770 kg/m^3. The fluid properties: density is 1000 kg/m^3, velocity of sound is 1450 m/s^2. The similar structure was considered in Reference 1. Table 4 shows the lowest three natural frequencies of shell (DYAS-3.0), which are compared with results of Reference 1.

Table 1. Natural frequencies (w, Hz) and quality criterion (χ)

Mode i	First way		Second way	
	w_i	c_i	w_i	c_i
1	0.9996	0.00175	0.9996	0,000112
2	1,426	0,00207	1,426	0,000217
3	1,847	0,00181	1,847	0,000313
4	2,305	0,00082	2,305	0,000462
5	2,818	0,00107	2,818	0,000513
6	3,379	0,00394	3,379	0,000777
7	3,939	0,00762	3,939	0,00103
8	4,343	0.01090	4,343	0.00128
9	4,910	0.00053	4,910	0,000065
Number of masters	120		10	

Table 4. Natural frequencies of hemispherical shell with water

	Mode 1	Mode 2	Mode 3
Reference 1	0,44	0,62	0,79
DYAS-3.0	0,43	0,61	0,78

The agreement with solution [1] is good.

Conclusion

The technique developed is an efficient and practical method for dynamic analysis of surface wave-fluid-structure systems. For eigenvalue problem solution the frequency condensation method is used. The features of frequency condensation method application to structure-fluid eigenvalue problem have been considered. The method has been validated by comparison with other solution.

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