Analysis of certain dynamic schemes of active-adaptive vibroinsulation

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1. The main idea of presented work is using the synchcronization of vibro-exciters phenomena for the active-adaptive suppression of rigid body oscillations.

One of the principal regularity of self-synchcronization phenomena is integral criterion of synchronous motions stability [1]. According to this criterion, a few unbalanced rotors installed in common oscillatory system rotate in synchronous mode with initial phases which provide the minimum of average value for Lagrangian of oscillating part of the system. In [2] it was shown that integral criterion could be extended to the systems with external synchronization. In the case when vibro-exciters are installed on the soft vibro-insulated rigid body, stable synchronous motions correspond to minima of average value of carrying body s kinetic energy $\left\langle T^{(\mathrm{I})} \right\rangle$ calculated in assumption of rotors uniform rotating in accordance with the law

$$\varphi_i = \omega t + \alpha_i \tag{1.1}$$

where α_i are initial phases.

Let us suppose that phase combination α_i , while $\left\langle T^{(\mathrm{I})} \right\rangle$ = 0, is possible. In this case the complete compensation of unbalanced forces and moments takes place , and, according to integral criterion, this phase combination is necessarily stable. Pointed phase combination is named compensating phase combination.

2. Earlier we had analyzed the simplest scheme of active-adaptive vibro-insulation presented in Fig.2.1. The harmonic constraining force $f \sin \omega t$ is imposed to the soft vibro-insulated carrying body. While operating, this force can change both in amplitude and phase. In static position the line of force action coincides with one of the principal central inertia axis (Oz).

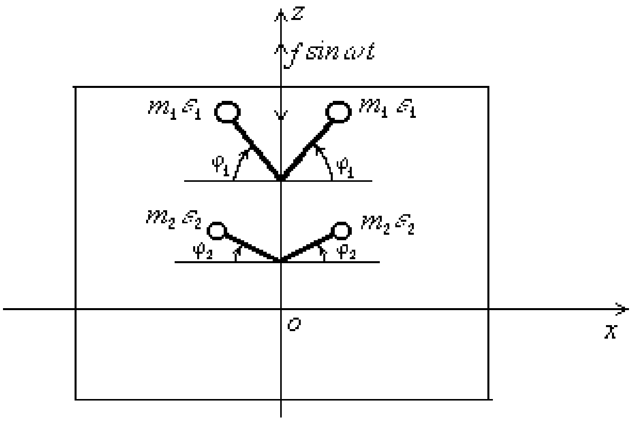

Fig.2.1

With the aim of constraining force suppression four unbalanced vibro-exciters activated by asynchronous motors are mounted on the carrying body. The vibro-exciters are connected kinematically in pairs so that each pair generates the constraining force acting in the same direction as the external force $f \sin \omega t$. Let us denote mass and eccentricity of the rotors of the first pair as m_1 and ε_1 , and mass and eccentricity of the rotors of the second one as m_2 and ε_2 . Stable synchronous motions respond to minima points of function

$$\left\langle T^{(\mathrm{I})} \right\rangle = \frac{\omega^2}{M}\left[s s_1 \cos \alpha_1 + s s_2 \cos \alpha_2 + 2 s_1 s_2 \cos(\alpha_1 - \alpha_2) \right] \tag{2.1}$$

where M is the mass of rigid body, $s = \frac{f}{\omega^2}$, $s_1 = m_1\varepsilon_1$, $s_2 = m_2\varepsilon_2$.

In the case when segments of the lengths s, $2s_1$ and $2s_2$ can form a triangle, provided by inequalities

$$s < 2s_1 + 2s_2 , \quad 2s_1 < s + 2s_2 , \quad 2s_2 < s + 2s_1 , \tag{2.2}$$

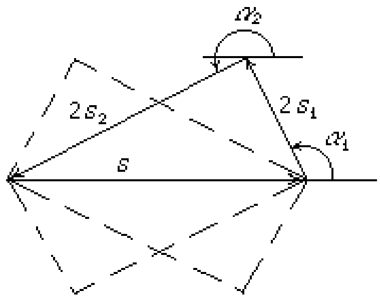

Fig.2.2

compensating phase combinations exist, and for these phase combinations the following relations take place

$$\begin{aligned} s + 2s_1\cos\alpha_1 + 2s_2\cos\alpha_2 = 0 \\ 2s_1\sin\alpha_1 + 2s_2\sin\alpha_2 = 0 \end{aligned} \tag{2.3}$$

In the system under consideration there are four compensating phase combinations at all. In accordance with integral criterion formulated above, compensating phase combinations are stable. It was also proved that the phase combinations of the second solution group determined by relations

$$\sin\alpha_1 = 0 , \quad \sin\alpha_2 = 0 \tag{2.4}$$

were unstable under fulfilling the conditions (2.2) . Obtained results are valid if partial velocities of vibro-exciters coincide with a constraining force frequency ω. However, if partial velocities differ slightly from ω, then the tendency of external constraining force suppression remains. Let us note that here as a partial velocity we mean the rotating velocity of vibro-exciter mounted on immovable base. So, according to mentioned adaptive property it is realized such phase combination α_1, α_2 which provide the complete suppression of carrying body oscillations.

It is to be noted that in the case when either damping or stiffness of rigid body support are not small the complete compensation of constraining action is also possible, and compensating phase combination (2.3) stability regions received under quasiconservative assumption [3] are more restricted.

3. In present section it is suggested the scheme different from the one presented in Fig.2.1. ,in the first, by vibro-exciters location and ,in the second, by no kinematic connection between them. (See Fig.3.1.)

Let Oxyz be the system of principal central inertia axes of rigid body. Harmonic force $f\sin\omega t$, oriented in the Oz direction, is applied to the body. With the purpose of oscillation suppression four unbalanced vibro-exciters activated by asynchronous motors are mounted on the carrying body. The rotors of the first and second vibro-exciters are rotating in opposite directions in the plane xOz, whereas the rotors of the third and fourth vibro-exciters are rotating in opposite directions in the plane yOz.

Taking into account (1.1), equations of soft vibro-insulated rigid body motions has a form

$$\begin{aligned} M\ddot{x} &= m_1\varepsilon_1\omega^2[\cos(\omega t + \alpha_2) - \cos(\omega t + \alpha_1)] \\ M\ddot{y} &= m_2\varepsilon_2\omega^2[\cos(\omega t + \alpha_4) - \cos(\omega t + \alpha_3)] \\ M\ddot{z} &= f\sin\omega t + m_1\varepsilon_1\omega^2[\sin(\omega t + \alpha_1) + \sin(\omega t + \alpha_2)] \\ &\quad + m_2\varepsilon_2\omega^2[\sin(\omega t + \alpha_3) + \sin(\omega t + \alpha_4)] \end{aligned} \tag{3.1}$$

$$\begin{aligned} I_x\ddot{\varphi}_x &= m_2\varepsilon_2\omega^2 a_2[\sin(\omega t + \alpha_4) - \sin(\omega t + \alpha_3)] \\ I_y\ddot{\varphi}_y &= -m_1\varepsilon_1\omega^2 a_1[\sin(\omega t + \alpha_2) - \sin(\omega t + \alpha_1)] \\ I_z\ddot{\varphi}_z &= 0 \end{aligned}$$

where M is the mass of rigid body, I_x, I_y, I_z are its principal central moments of inertia; x, y, z, ϑ_x, ϑ_y, ϑ_z are coordinates of the center of body s masses and its angles of turning about the axes Ox, Oy, Oz; m_1 and ε_1 are mass and eccentricity of the first and second vibro-exciters, m_2 and ε_2 are mass and eccentricity of the third and fourth vibro-exciters; a_1 and a_2 are the distances center of masses O from the axes of vibro-exciters rotating.(See Fig 3.1.)

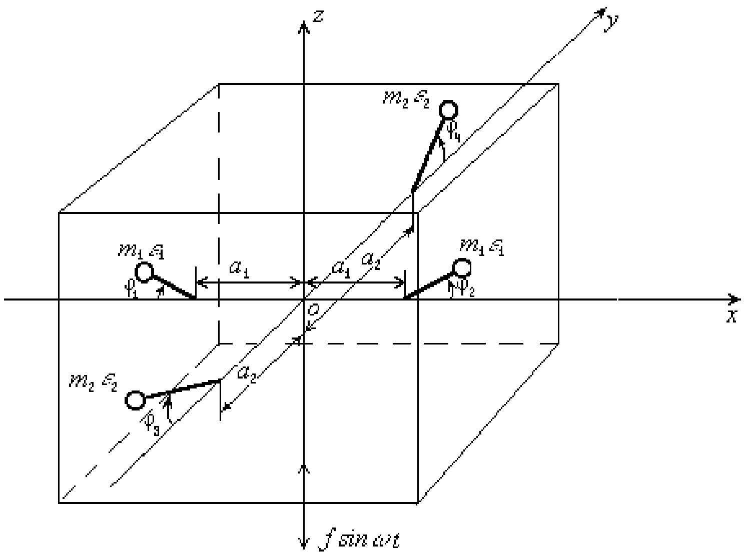

Fig.3.1

Expression for averaged kinetic energy of rigid body has a form

$$\left\langle T^{(I)}\right\rangle = \frac{f}{2M}\left[m_1\varepsilon_1(\cos\alpha_1+\cos\alpha_2)+m_2\varepsilon_2(\cos\alpha_3+\cos\alpha_4)\right]+$$
$$+\frac{m_1\varepsilon_1 m_2\varepsilon_2\omega^2}{M}\left[\cos(\alpha_1-\alpha_3)+\cos(\alpha_2-\alpha_3)+\cos(\alpha_1-\alpha_4)+\cos(\alpha_2-\alpha_4)\right]-$$
$$-\frac{1}{2I_y}(m_1\varepsilon_1\omega a_1)^2\cos(\alpha_1-\alpha_2)-\frac{1}{2I_x}(m_2\varepsilon_2\omega a_2)^2\cos(\alpha_3-\alpha_4) \qquad (3.2)$$

The initial phases α_1, α_2, α_3, α_4 meet the system of transcendental equations

$$-\frac{1}{s_1}\frac{\partial\left\langle T^{(I)}\right\rangle}{\partial\alpha_1}\equiv s\sin\alpha_1+s_2\sin(\alpha_1-\alpha_3)+s_2\sin(\alpha_1-\alpha_4)-s_1A\sin(\alpha_1-\alpha_2)=0$$
$$-\frac{1}{s_1}\frac{\partial\left\langle T^{(I)}\right\rangle}{\partial\alpha_2}\equiv s\sin\alpha_2+s_2\sin(\alpha_2-\alpha_3)+s_2\sin(\alpha_2-\alpha_4)-s_1A\sin(\alpha_2-\alpha_1)=0$$
$$-\frac{1}{s_2}\frac{\partial\left\langle T^{(I)}\right\rangle}{\partial\alpha_3}\equiv s\sin\alpha_3+s_1\sin(\alpha_3-\alpha_1)+s_1\sin(\alpha_3-\alpha_2)-s_2B\sin(\alpha_3-\alpha_4)=0 \qquad (3.3)$$
$$-\frac{1}{s_2}\frac{\partial\left\langle T^{(I)}\right\rangle}{\partial\alpha_4}\equiv s\sin\alpha_4+s_1\sin(\alpha_4-\alpha_1)+s_1\sin(\alpha_4-\alpha_2)-s_2B\sin(\alpha_4-\alpha_3)=0$$

Here the following notations are introduced:

$$s = \frac{f}{\omega^2} \quad , \quad s_1 = m_1\varepsilon_1 \quad , \quad s_2 = m_2\varepsilon_2 \quad , \quad A = \frac{Ma_1^2}{I_y} \quad , \quad B = \frac{Ma_2^2}{I_x} \tag{3.4}$$

The solution of the system (3.3) consists of the following groups.

I.

$$\begin{aligned} & s + 2s_1 \cos\alpha_1 + 2s_2 \cos\alpha_3 = 0 \\ & 2s_1 \sin\alpha_1 + 2s_2 \sin\alpha_3 = 0 \\ & \alpha_2 = \alpha_1 \\ & \alpha_4 = \alpha_3 \end{aligned} \tag{3.5}$$

II.

$$\alpha_i = \pi n_i \ , \ i = 1,\ 2,\ 3,\ 4 \tag{3.6}$$

where each number n_i can take values of 0 or 1.

III. a)

$$\begin{aligned} & \alpha_1 = 0 \\ & \alpha_2 = 0 \\ & \alpha_4 = -\alpha_3 \\ & \cos\ \alpha_3 = \frac{s + 2s_1}{2Bs_2} \end{aligned} \tag{3.7}$$

b)

$$\begin{aligned} & \alpha_1 = 0 \\ & \alpha_2 = \pi \\ & \alpha_4 = -\alpha_3 \\ & \cos\alpha_3 = \frac{s}{2Bs_2} \end{aligned}$$

c)

$$\begin{aligned} & \alpha_1 = \pi \\ & \alpha_2 = 0 \\ & \alpha_4 = -\alpha_3 \\ & \cos\alpha_3 = \frac{s}{2Bs_2} \end{aligned} \tag{3.8}$$

d)

$$\begin{aligned} & \alpha_1 = \pi \\ & \alpha_2 = \pi \\ & \alpha_4 = -\alpha_3 \\ & \cos\ \alpha_3 = \frac{s - 2s_1}{2Bs_2} \end{aligned} \tag{3.9}$$

IV. This group of solutions is received from the group III by changing the indexes 1<->3, 2<->4 and, simultaneously, by changing A<->B.

V.

$$\begin{aligned} & \alpha_2 = -\alpha_1 \\ & \alpha_4 = -\alpha_3 \\ & s + 2s_2 \cos\alpha_3 - 2As_1 \cos\alpha_1 = 0 \\ & s + 2s_1 \cos\alpha_1 - 2Bs_2 \cos\alpha_3 = 0 \end{aligned} \tag{3.10}$$

As in the case of the scheme described in section 2, we assume that the inequalities (2.2) are satisfied. Then the first group of solutions responds to regime when the oscillations of the carrying body are absent. In specified sense, the first group of solutions corresponds to compensating phase combinations of the system considered in the section 2. (See Fig.2.2.) However, in general case among the solutions (3.6) - (3.10) the stable phase combinations, which do not provide full suppression of body s oscillations, may occur. For example, if A=B=1 and $s_2 = 0.5\, s_1$, then direct numerical testing shows that in the range $0 < s < s_1$ the phase combination $\alpha_1 = \alpha_2 = \pi$, $\alpha_3 = \alpha_4 = 0$ is also stable. Belonging the second group, this phase combination produces the body s oscillations in direction of axis Oz. It should be noted that here we consider the stability conditions following from the demand of positive definiteness of matrix $\left\| \frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_i \partial \alpha_j} \right\|$, where $\langle T^{(I)} \rangle$ is determined by (3.2). These conditions of stability correspond to nonquasiconservative statement of the problem on synchronization [1].

Let us show that there are parameters of active-adaptive vibration suppressor presented in Fig. 3.1 while unique stable phase combination is the compensating one (3.5). Let the range of constraining force vibration amplitude be

$$0 \le f \le f_0 \tag{3.11}$$

We can choose the parameters of the suppressor as

$$s_1 = s_2 = \frac{f_0}{4\omega^2} \quad , \quad a_1 = \sqrt{\frac{I_y}{M}} \quad , \quad a_2 = \sqrt{\frac{I_x}{M}} \tag{3.12}$$

It should be noted that, due to the theory of synchronization, the magnitude of $\frac{m_1}{M}$ is considered as a small parameter; therefore, the magnitude of f_0 has upper limit determined by parameters of carrying system. From (3.12) it follows that phase combinations of V group exist only in the case when $s = 0$. So, it remains to show that, while fulfilling relations (3.11) and (3.12), the phase combinations of II and III groups are unstable. The case of IV solution group is completely similar to the case of III group. For the given system the elements of the matrix of second-order derivatives $\left\| \frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_i \partial \alpha_j} \right\|$ are calculated by the formulas

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1^2} = s_1 \left[-S \cos \alpha_1 + s_1 + 2 s_1 \cos(\alpha_1 - \alpha_2) \right]$$

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_2^2} = s_1 \left[-S \cos \alpha_2 + s_1 + 2 s_1 \cos(\alpha_1 - \alpha_2) \right]$$

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_3^2} = s_1 \left[-S \cos \alpha_3 + s_1 + 2 s_1 \cos(\alpha_3 - \alpha_4) \right]$$

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_4^2} = s_1 \left[-S \cos \alpha_4 + s_1 + 2 s_1 \cos(\alpha_3 - \alpha_4) \right]$$

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1 \partial \alpha_2} = -s_1^2 \cos(\alpha_1 - \alpha_2) \qquad \frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1 \partial \alpha_3} = s_1^2 \cos(\alpha_1 - \alpha_3)$$

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1 \partial \alpha_4} = s_1^2 \cos(\alpha_1 - \alpha_4) \qquad \frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_2 \partial \alpha_3} = s_1^2 \cos(\alpha_2 - \alpha_3)$$

$$\frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_2 \partial \alpha_4} = s_1^2 \cos(\alpha_2 - \alpha_4) \qquad \frac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_3 \partial \alpha_4} = -s_1^2 \cos(\alpha_3 - \alpha_4)$$

where $S = s + s_1(\cos \alpha_1 + \cos \alpha_2) + s_2(\cos \alpha_3 + \cos \alpha_4)$.

For the proof of instability it is sufficient to show that at least one of the principal minors of the matrix $\left\| \dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_i \partial \alpha_j} \right\|$ is negative.

Solutions of the second group consists of six basic variants :

1) $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$; $\Omega = \omega + 4\omega_1$, $\dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1^2} = -(\omega + \omega_1)\omega_1 < 0$;

2) $\alpha_1 = \pi, \quad \alpha_2 = \alpha_3 = \alpha_4 = 0$; $\Omega = \omega + 2\omega_1$, $\dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_2^2} = -(\omega + 3\omega_1)\omega_1 < 0$;

3) $\alpha_1 = \alpha_2 = \pi, \quad \alpha_3 = \alpha_4 = 0$; $\Omega = \omega$, $\Delta = \left| \dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_i \partial \alpha_j} \right| = \omega^2 \omega_1^4 (\omega - 4\omega_1)(\omega + 4\omega_1) < 0$,

as $\omega < 4\omega_1$ due to (3.4), (3.11), (3.12).

4) $\alpha_1 = \alpha_3 = \pi; \quad \alpha_2 = \alpha_4 = 0$; $\Omega = \omega$, $\dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_2^2} = -(\omega + \omega_1)\omega_1 < 0$;

5) $\alpha_1 = \alpha_2 = \alpha_3 = \pi; \quad \alpha_4 = 0$; $\Omega = \omega - 2\omega_1$, $\Delta = \left| \dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_i \partial \alpha_j} \right| = -\omega\omega_1^4 (\omega - 2\omega_1)^2 (2\omega_1 + \omega) < 0$;

6) $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \pi$; $\Omega = \omega - 4\omega_1$, $\Delta = \left| \dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_i \partial \alpha_j} \right| = \omega^3 \omega_1^4 (\omega - 4\omega_1) < 0$,

The proof of instability for the rest solutions of the II group is analogous to one of mentioned. It is to be noted that cases

$$\omega = 2\omega_1 \ , \quad \omega = 4\omega_1 \qquad (3.13)$$

are of singular character; so these cases require, generally speaking, additional theoretical investigation. However, from the practical point of view , mentioned cases are not of interest because of their realization needs the constant value of constraining force amplitude. But if amplitude of constraining force does not change, while operating, the parameter ω_1 is surely to be chosen so that equalities (3.13) are not fulfilled.

Let us consider solution group III .

In case a) - (3.7)

$\alpha_1 = \alpha_2 = 0$, $\alpha_4 = -\alpha_3$, $\cos\alpha_3 = \dfrac{\omega + 2\omega_1}{2\omega_1}$, $\Omega = 2(\omega + 2\omega_1)$, $\dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1^2} = -\omega_1(2\omega + \omega_1) < 0$

In case b) - (3.8)

$\alpha_1 = 0$, $\alpha_2 = \pi$, $\alpha_4 = -\alpha_3$, $\cos\alpha_3 = \dfrac{\omega}{2\omega_1}$, $\Omega = 2\omega$, $\dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_1^2} = -\omega_1(2\omega + \omega_1) < 0$

In case c) - (3.8)

$\alpha_1 = \pi$, $\alpha_2 = 0$, $\alpha_4 = -\alpha_3$, $\cos\alpha_3 = \dfrac{\omega}{2\omega_1}$, $\Omega = 2\omega$, $\dfrac{\partial^2 \langle T^{(I)} \rangle}{\partial \alpha_2^2} = -\omega_1(2\omega + \omega_1) < 0$

In case d) - (3.9)

$\alpha_1 = \pi$, $\alpha_2 = \pi$, $\alpha_4 = -\alpha_3$, $\cos\alpha_3 = \dfrac{\omega - 2\omega_1}{2\omega_1}$, $\Omega = 2(\omega - 2\omega_1)$, $\dfrac{\partial^2 \langle T \rangle}{\partial \alpha_3^2} = -\omega_1^2 < 0$

So, choosing parameters of active-adaptive system satisfying condition (3.12), one can provide the complete suppression of carrying body oscillations. This suppression will be adaptive with respect to external action in the range of constraining force amplitudes (3.11).

The absence of kinematic connection between the vibro-exciter rotors simplifies the realization of the system of active-adaptive vibro-insulation and increases its reliability.

References

1. Blekhman , I.I. Synchronization in science and technology. 1988, New York : ASME Press.
2. Malakhova , O.Z.. On certain modification of extreme stability criterion for synchronous movements (in Russian) , *Izv. RAN. MTT (Mechanics of Solids),* 1995(5), pp.51-56.
3. Hagaev, R.F., Guzev, V.V. Self-synchronization of inertial vibro-exciters. 1990, Leningrad: Mashinostroenie. (in Russian)