Probabilistic approach to calculation of kinetics of crack meshes

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One of specific fracture mechanism of heat-stressed structures is formation and growth of systems (meshes) of surface cracks with subsequent crumbling. The problem of the analysis of crack mesh kinetics was considered in work [1, 2, 3] in deterministic formulation: a material of a structure and loading conditions were assumed uniform (homogenous), and crack mesh periodical. Such approach has allowed to reveal and approximately describe the mesh kinetics: formation in the beginning of short and close located cracks, then their growth, stop of some cracks and formation of a mesh of deeper and more distant cracks. In real structures however the growth of crack meshes is random process, and probabilistic approach is required for its analysis. The method of analysis must allow to determine the probability of occurrence of rather close and deep cracks and thus to estimate the opportunity of crumbling of the structure surface, which is usually more dangerous than just the development of a crack mesh.

The growth of crack mesh is probabilistic process because of two reasons: 1) inhomogeneity of a material, connected with its crystal structure and with variations of composition and structure of it along cross-section of a detail, and 2) non-uniformity of loading conditions caused by, for example, variations of convective film coefficient at the structure's surface due to it non-smoothness and other similar reasons. There are many works devoted to the analysis of a single crack growth at such conditions see, for example, [4]. For crack systems the probabilistic solving of the problem is usually limited to determination of possibility of start of the most dangerous crack; the material is assumed inhomogeneous, cracks are random, but the interaction of cracks is not considered [5].

In the given work the growth of random crack mesh is modelled with interaction of cracks taken into account.

As a methodical example let consider a cyclic thermal loading of halfspace of a linear-elastic brittle material. For the sake of simplicity (keeping the main peculiarity of the task the interaction of cracks) let allow that inhomogeneity of the material result to forming of random system of initial short cracks, while the process of growth of longer cracks can be described under usual hypothesis of continuous homogenous solids. Let also consider a temperature field uniform temperature depends only on distance up to a surface of half-space.

Let in a detail at some moment of time a crack mesh of various (random) lengths exist see fig. 1. We shall assume that the interaction of stress fields is essential only for the neighboring cracks, and the influence of farther cracks can be neglected. In a case shown on fig. 1 it means that at calculation of stress intensity factor at tip of crack l_2 it is enough to take into account influence of cracks' l_1 and l_3 only (exception those cases when the neighboring crack is much shorter than considered; numerical experiments show, that influence of cracks, twice shorter than considered(examined), in general it is possible to neglect irrespectively of distance up to them).

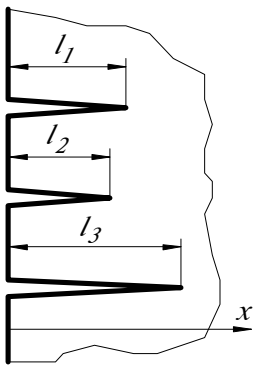

Fig. 1. Crack mesh (cross-section)

With such assumption and at the known history of temperature in a cycle $T(x,t)$, for any triads of neighboring cracks (l_1, l_2, l_3) stress intensity factor at the tip of middle crack of triad $K_1^{\langle 2\rangle}(t)$ can be calculated; which allow to determine the crack length increment per

cycle $\delta_2(l_1, l_2, l_3)$ using for example the Paris formulae. At non-critical growth of the cracks it is possible to neglect the influence of increments δ_1 and δ_3 on the calculation of δ_2 in a given cycle while $\delta_1 << l_1$ and $\delta_3 << l_3$ the initial values of l_1, l_3 can be used.

Let now at some cycle number N the probability of finding out of a crack triad (l_1, l_2, l_3) is equal to $p(l_1, l_2, l_3)$. Then after the following cycle the probability to find out a crack of length l^* can be expressed by the formula

$$p^* = \iint_{\Omega} p(l_1, l_2, l_3) d\Omega \tag{1}$$

where Ω a surface in space (l_1, l_2, l_3) satisfying to the equation

$$l_2 + \delta_2(l_1, l_2, l_3) = l^* \tag{2}$$

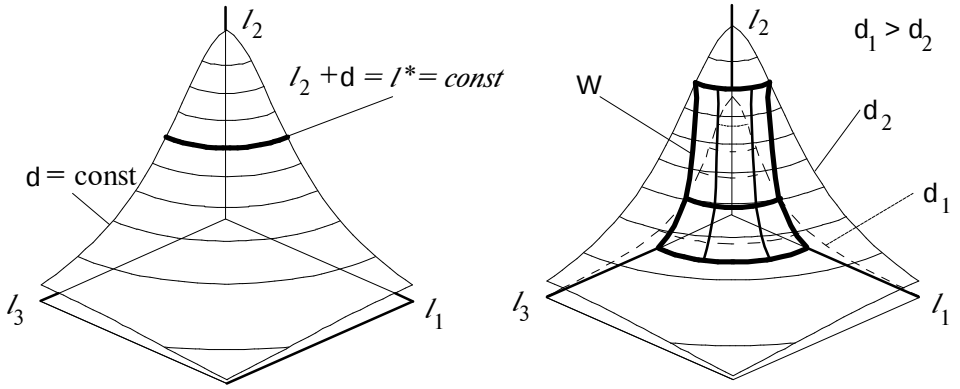

Fig. 2. To determination of a surface Ω

The $p^*(l)$ represents probability of occurrence of crack's triad where the middle one has the length l_2^* while l_1 and l_2 are arbitrary. Probability that the crack l_1 in the triad after the N-th cycle will be of the size l_1^* is

$$p_1^* = \int_{\Gamma} p(l_1, l_2, l_3) d\Gamma \tag{3}$$

where Γ the line on a plane $l_2 = l_2^*$ that satisfy to condition $l_1 + \delta(l_m, l_1, l_n) = l_1^*$ ($\delta(l_m, l_1, l_n)$ increment of a middle crack in triad (l_m, l_1, l_n) is determined similarly to δ_2 earlier).

Taking into account symmetry of a problem on rearrangement of indexes 1 and 3, for probability of occurrence of the triad (l_1^*, l_2^*, l_3^*) after N-th cycle we shall finally receive

$$p_{N+1}(l_1^*, l_2^*, l_3^*) = \iint_{\Omega} p_N(l_1, l_2, l_3) \left[\int_{\Gamma} p_N(l_m, l_1, l_n) d\Gamma \right]^2 d\Omega \tag{4}$$

The distance between cracks influences the calculation of p_{N+1} only as a parameter that affects the stress intensity factors and hence the form and sizes of the surfaces Ω and the contour Γ. As the distances between cracks does not vary during their growth, the calculations for different distances can be fulfilled independently.

The equation (4) was used for calculation of crack mesh in elastic half-space, loaded with a temperature field cyclically varied from uniform heating up to the

$$T(x) = T_0\left[1 - \exp(x/b)\right] \tag{5}$$

(parameter b does not vary during loading process). The calculation of stress intensity factor $K_1^{\langle 2\rangle}(l_1, l_2, l_3)$ was made numerically using FEA-package COSMOS/M, some results are shown on fig. 3.

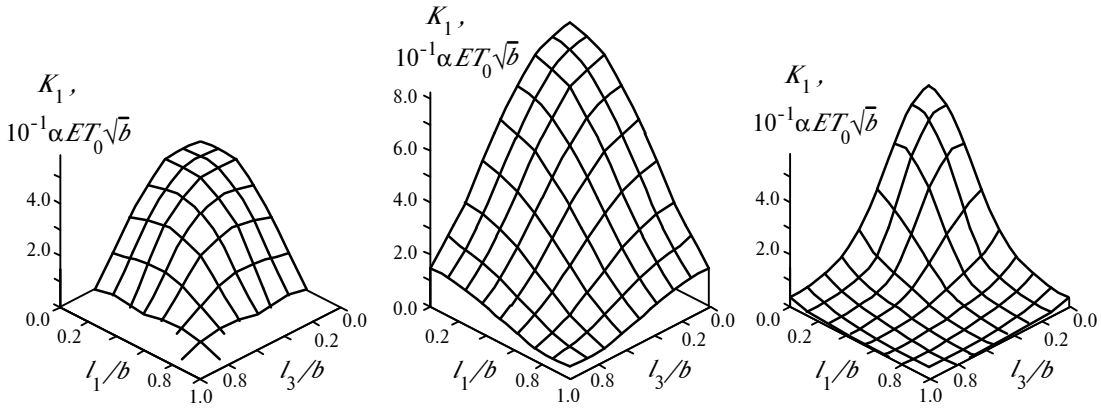

Fig. 3. Stress intensity factor at tip of a crack l_2
a) $l_2 = 0.5b$, $c = 0.5b$; b) $l_2 = 0.5b$, $c = b$; c) $l_2 = 0.25b$, $c = 0.5b$

Received stress intensity factor was used for calculation of crack length increments, wich are necessary for construction of the surface Ω. Under calculation it was assumed that the growth of a crack at cyclic loading is stated by the Paris formula:

$$\frac{\Delta l}{\Delta N} = C\Delta K_1^m \quad . \tag{6}$$

Now the distribution of triads of crack length can be figured out with the aid of the formulas (3) and (4). The more simple formula (3) can be applied only on the last step; all the intermediate steps require calculation of complete distribution with the formula (4); the main difficulty of which is the determination of the surface Ω. The calculation proceed with a statistical distribution of the lengths of crack for given distance between them for a given moment of a loading history (or with use (3) distribution of cracks' triads). For various distances between cracks the results will appear various, as the more distant cracks influence each other more weakly compare fig. 3a and 3б.

Example of results, received for considered model problem, are shown on fig. 4, where lines of equal value show the probability of finding cracks of given length l at distance between cracks c. Comparison with results of measurement of cracks in a real structure (cast-iron mould fig. 4б) shows qualitative similarity of results. It is necessary to note, however, that obtaining of probabilistic estimations appear much more complex than deterministic mainly because of difficulties in determination of a surface Ω.

Numerical experiments, simulating growth of a crack mesh with uniform initial distributions of the parameters l and c ($l \in [l_1, l_2]$ $c \in [c_1, c_2]$) show, that the parameters of a crack mesh to a moment of its stop weakly depend on meanings of l_1, l_2, c_1, c_2 if these sizes are rather small. In some way it can be treated as a proof of opportunity to choose an arbitrary initial crack mesh at calculation of the mesh kinetics in deterministic formulation, if the initial cracks and distances between them are small enough. However strict proof of this fact as far as any direct estimations of probabilistic parameters of received meshes are still not found.

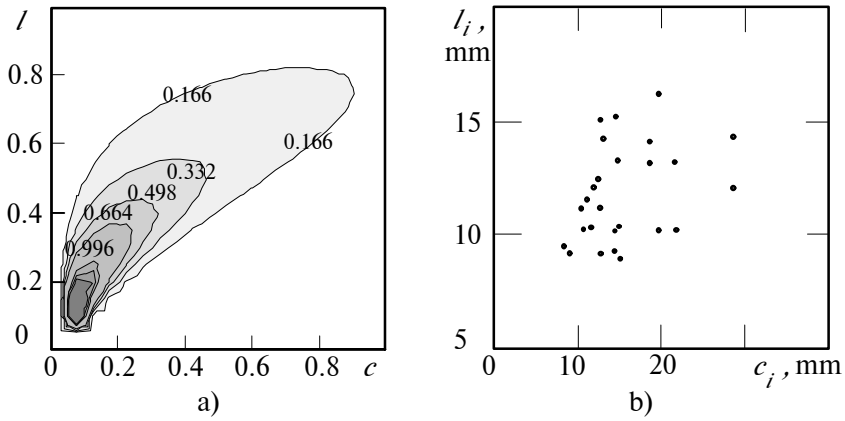

Fig. 4. Distribution of lengths of cracks. a - model problem, b - real structure

It is interesting to compare received results with direct modeling of mesh kinetics worked out by Monte-Carlo method. A similar elastic half-plane with initial cracks was considered; lengths of cracks l_i and the distances between the next cracks c_i in an initial moment of time are random with uniform distribution $l \in [0.1b; 0.3b]$ and $c \in [0.1b; 0.3b]$ (without corellation). It was accepted, that the characteristics of material and loading parameters (see the formulas 5-6) are connected by the following dependences: $K_{1c} = 10\mathrm{a}ET_0\sqrt{b}$, $K_{th} = 10^{-4}\mathrm{a}ET_0\sqrt{b}$, $C = 0.1b$, $m = 4$.

For reception of a stable results at use of Monte-Carlo method it is necessary to consider rather large number of cracks (~10^3 ..10^4) because the calculated distribution is two-dimensional (plane c-l) and the fact that significant part of cracks stoped to growth during loading process. Thus, of course, the exact computation of stress intensity factor at cracks' tips practically cannot be carried out; but for exponential temperature field like (5) estimations similar to represented on fig. 3 can be applied.

The results of modeling are shown on fig. 5, which show images of crack mesh for a number of loading cycles in coordinates "length of cracks l distance between them c" (left) and "length of cracks l distance along half-plane edge y" (right, these last drawings are compressed in a direction of the y-axis). They qualitatively confirm results received earlier at the solution of a problem in deterministic formulation [2] gradual stop of some cracks with increasing of length of the deepest omes. Simplified calculation of stress intensity factor allows to receive some quantitative estimations, though required for this purpose large number of cracks demands rather large time to compute. So, receiving of fig. 5 on i486DX4-100 takes about 20 mines of time of calculations, though the number of cracks considered (10^3) is not enough for obtaining of valid estimations of probability for long ($l > 1.5b$) cracks and the simpliest structure half-space allow not to use FEA for calculations of strain energy in part of a detail, distant from cracks.

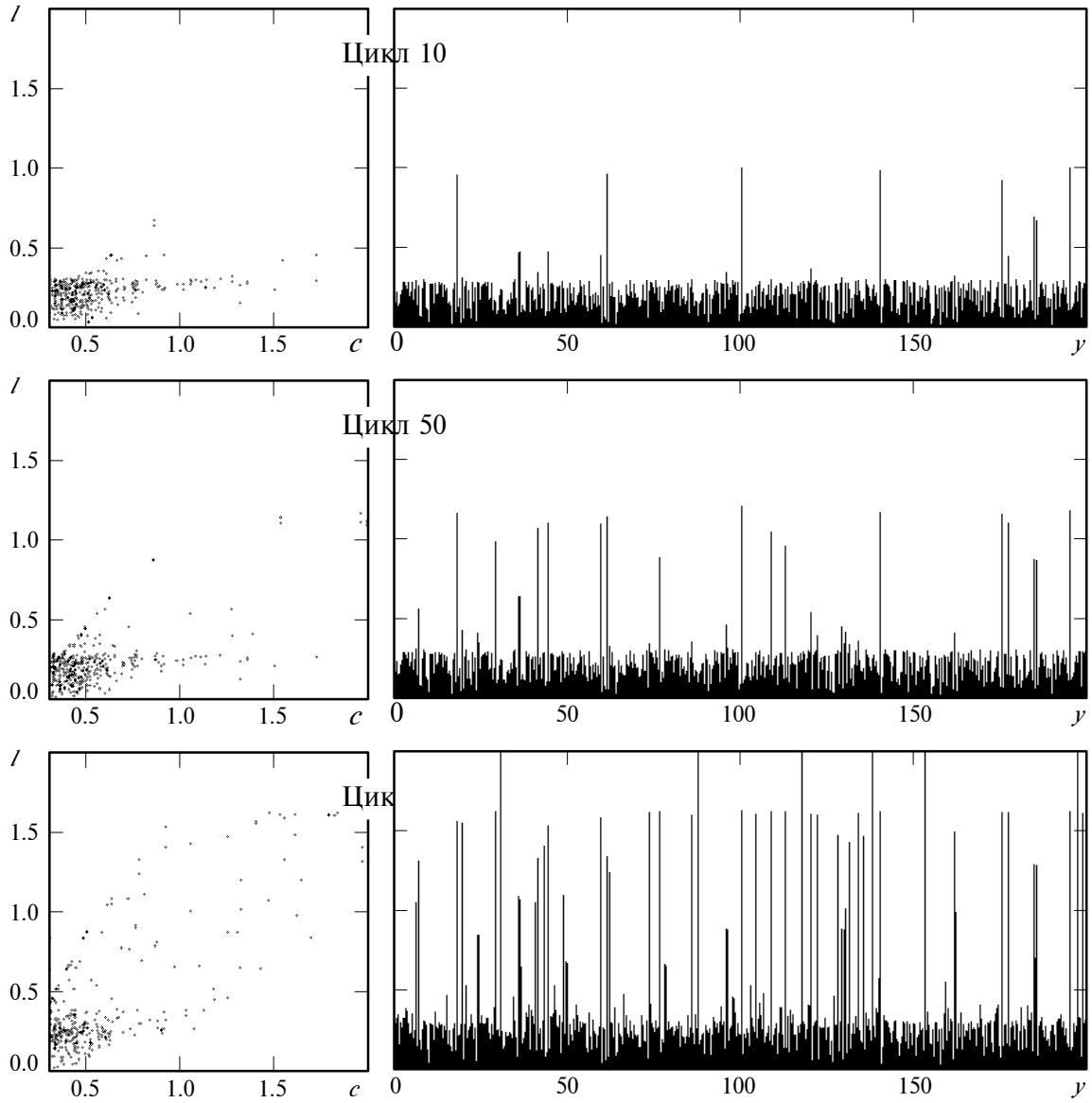

Fig. 5. Modeling growth of a grid of cracks (method Monte-Carlo)

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