On the ultimate strain criterion for fracture prediction at normal and elevated temperatures

D.A.Gokhfeld, O.S.Sadakov and K.M.Kononov

Cheliabinsk State Technical University, Russia

Majority of the known fracture criteria for combine stress state are formulated proceeding from the stress tensor invariant characteristics (see, for example, survey in Kachanov's book [1] and the reference book [2]). In this paper criterion of another, deformation type is considered. It embraces quasistatical one-fold loading conditions at normal and elevated temperatures, i.e. conditions of short-term and long-term fractures. The new criterion is based on the experimental data which were obtained by Bridgman [3], Kolmogorov [4] and some other investigators. It seems, that criteria of deformation type are, in general, more adequate from the physical point of view. Correspondence of the suggested criterion with some known fracture theories and experimental data is discussed below.

1. Fracture of materials is considered now as a kinetical process consisting of the following main stages.

a) Damage accumulation in micro-volumes of a body. As it is supposed, the damages are caused by inelastic micro-strains and thus they are related with the shear stresses.

b) Appearance of fracture surfaces resulting from micro-cracks in vicinities of the dislocation intersects, micro-pores on grain boundaries and other defects; merges of micro-cracks and formation of macro-cracks.

To describe the initial stages of fracture usually a damage function is introduced. Violation of the body continuity is supposedly considered as corresponding to the moment when the introduced damage function reaches its critical value.

c) Macro-cracks propagation which occurs in metals either across the grains or along their boundaries and lasts until the body becomes divided into parts. Depending on character of the process, two main types of fracture (the brittle and the viscous ones) are usually distinguished. Crack growth at the brittle fracture occurs at elastic state of material while the viscous one is accompanied by macro-plastic deformation. Actual character of the realizing fracture depends on properties of the material and on loading conditions (strain rate, temperature, the type of stress state and others).

However, as the investigations show, the two first stages of fracture which precede formation of a small crack are almost independent (for metallic alloys and some other materials) on the loading state and quality of materials: in any case these stages are related with inelastic strains.

Prediction of the conditions at which local crack can appear is just the subject of the strength theories. In overwhelming majority of the latter the corresponding criterion is postulated in terms of the stress tensor invariants. Bearing in mind the following comparisons with the suggested criterion, let us remind fracture conditions which follow from the three strength theories which are the most widely used in science and applications (see [1,2,5,6]:

$$A\tau_{max} + B\sigma_{\tau\,max} = \sigma_F \quad \text{(Mohr-Guest);} \qquad (1)$$

$$C\tau_0 + D\sigma_0 = \sigma_F \quad \text{(Nadai-Schleicher);} \qquad (2)$$

$$G\sigma_e + H\sigma_1 = \sigma_F \quad \text{(Pysarenko-Lebedev).} \qquad (3)$$

Here

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}, \quad \sigma_{\tau\,max} = \frac{\sigma_1 + \sigma_3}{2} \qquad (4)$$

- shear and normal stresses in a surface element where the former one has its maximum value;

$$\tau_0 = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2},$$
$$\sigma_0 = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) \tag{5}$$

- stresses in the octahedral surface element;

$$\sigma_e = \frac{3}{\sqrt{2}}\tau_0 \tag{6}$$

- stress intensity; $\sigma_1 \geq \sigma_2 \geq \sigma_3$ - the principal stresses; σ_F - true tensile strength.

For the left-hand sides of exps (1)-(3) usually terms "equivalent" or "effective" stresses (σ_{ef}) are employed. Any of the expessions includes two constants (A, B etc) which can be determined by making use experimental values of the ultimate stresses specific for some two types of tests, e.g. simple tension (σ_F) and compression (σ_F^c) or tension and torsion (τ_F). Thus, the fracture conditions (1)-(3) can be accordingly presented in the following forms:

$$\sigma_1 - \nu\sigma_3 = \sigma_F; \quad \nu = \sigma_F/\sigma_F^c; \qquad \text{(Mohr)} \tag{7a}$$

$$\sigma_1 - (\kappa-1) = \sigma_F; \quad \kappa = \sigma_F/\tau_F; \qquad \text{(Guest)} \tag{7b}$$

$$\sqrt{\frac{3}{2}}\kappa\tau_0 - \sqrt{3}\left(\kappa-\sqrt{3}\right)\sigma_0 = \sigma_F, \quad \kappa = \sigma_F/\tau_F; \tag{8}$$

$$\nu\sigma_e + (1-\nu)\sigma_1 = \sigma_F \tag{9}$$

(version (7b) given by Guest permits to avoid difficulties related with determination of ultimate stress under compression for ductile materials).

Hence, corresponding relations between the three basic characteristics of strength (σ_F, σ_F^c, τ_F) follow:

$$\tau_F = \frac{\sigma_F}{1+\nu} = \frac{\sigma_F\sigma_F^c}{\sigma_F+\sigma_F^c}; \tag{10}$$

$$\tau_F = \frac{2\sigma_F}{1+\nu\sqrt{3}} = \frac{2}{\sqrt{3}}\cdot\frac{\sigma_F\sigma_F^c}{\sigma_F+\sigma_F^c}; \tag{11}$$

$$\tau_F = \frac{\sigma_F}{1+\nu\left(\sqrt{3}-1\right)} = \frac{\sigma_F\sigma_F^c}{\sigma_F\left(\sqrt{3}-1\right)+\sigma_F^c}; \tag{12}$$

These relations can be used for partial experimental verification of the mentioned strength theories.

2. However, there exist some strength theories which are based on strain criteria. First of them is the maximum strain theory by which constant ultimate strain is postulated to be independent on the stress state (Poncelet, Saint-Venant).The theory was generally accepted in the past but now is not used as it contradicts experimental data. Further investigations carried out by Bridgman [3], Kolmogorov [4] and other authors have shown that the ultimate strain of any material depends essentially on the type of stress state. To characterize the latter an index (factor) $\beta = \sigma_0/\sigma_e$ can be used.

Fig.1 illustrates the corresponding regularity which was obtained by tests and is typical for alloys. The ordinates present relative values of inelastic strain intensity

$$p_e = \frac{\sqrt{2}}{3}\left[(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2\right]^{1/2} = \frac{1}{\sqrt{2}}\gamma_{0p} \qquad (13)$$

which correspond to fracture conditions; the latter are refered to that for uniaxial tension (p_F). Here p_1, p_2, p_3 are principal inelastic strain components. It should be noted that initially the Odquist's parameter was used here as ordinate of the diagram [4] however introduction of the above adopted relative coordinate seems to be more adequate: the corresponding curves constructed for different materials prove to be often practically coincident. It should be stressed that here the true ultimate inelastic tensile strain (by which necking of the specimen under tension is taken into account) is meant; the latter is determined by expression

$$p_F = \ln\frac{1}{1-\psi}, \qquad \psi = \frac{S_0 - S_F}{S_0}; \qquad (14)$$

ψ- is relative contraction of specimen's cross-sectional area S.

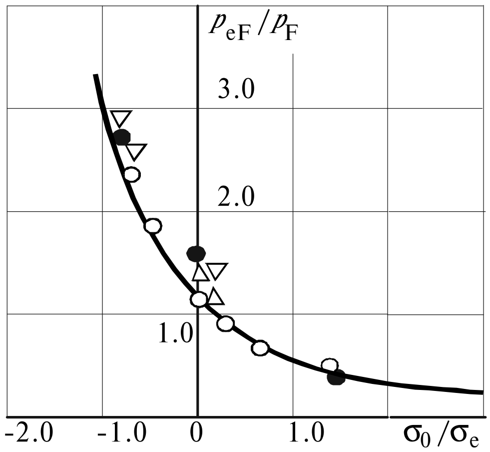

Fig. 1. The relative ultimate plastic strain versus an index which characterizes the type of stress state (o- cast low-carbon steels; Δ- cupper alloy; •- nickel alloy; ∇- aluminium alloy).

3. Function

$$p_{eF}/p_F = \varphi(\beta), \quad (\beta = \sigma_0/\sigma_e) \qquad (15)$$

by which ultimate strains at arbitrary stress state is determined, should satisfy the following conditions:

a) under uniaxial tension - $\beta = 1/3$, $p_{eF}/p_F = 1$;

b) under torsion - $\beta = 0$, $p_{eF}/p_F = \gamma_F/(p_F\sqrt{3})$;

c) under uniaxial compression - $\beta = -1/3$, $p_{eF}/p_F = p_F^c/p_F$;

(p_F^c is true ultimate strain under compression);

d) at equal tri-axial tension - $\beta = \infty$, $p_{eF} = 0$,
(fracture is possible but it is not conditioned by plastic strains);

e) at any stress state which is superposed by hydrostatic compression the latter prevents appearance of cracks;thus the breaking inelastic strain grows unlimitedly with the pressure increasing.

Analysis shows that the ultimate plasticity function (15) can be approximated by an exponent:

$$p_{eF}/p_F = a\exp(\beta b) \qquad (16)$$

Parameters a,b should be found for any material by making use the test data obtained at uniaxial tension and alternatively torsion or uniaxial compression.

So it can be got

$$a = \gamma_F/(p_F\sqrt{3}), \quad b = -3\ln a. \qquad (17)$$

4. However, application of any strength theory is more convenient when the latter is formulated in terms of stress state directly (13). The proper transformation can be easily performed assuming that at proportional loading conditions the generalized stress-strain relation

is invariant with respect to the type of stress-strain state and can be approximated by a power function

$$\sigma_e = Kp_e^m \tag{18}$$

with rather good correspondence for metallic alloys and some other materials. Variables σ_e, p_e are defined by eqns (5),(6), (13) while coefficient K and hardening index m are to be determined for any material using its actual stress-strain diagram. Employing the adopted approximation, the ultimate plasticity function (16) can be presented in the form

$$\sigma_{eF}/\sigma_F = \left(a\exp(\beta b)\right)^m . \tag{19}$$

Parameter a expressed in terms of ultimate shear stress and tensile strength becomes equal to

$$a = \left(\tau_F\sqrt{3}/\sigma_F\right)^{1/m}; \tag{20}$$

alternative expression can be obtained if the compression strength is used instead of τ_F:

$$a^{2m} = \sigma_F^c/\sigma_F = 1/\nu . \tag{21}$$

Excluding parameter a from exps (21), (22) we come to relation between the three breaking stresses which follows from the deformation criterion:

$$3\tau_F^2 = \sigma_F\sigma_F^c . \tag{22}$$

It should be reminded that the true stresses σ_F and σ_F^c are here meant.

Relations (19), (17) lead to definition of the effective stress accordingly to the criterion which is considered

$$\sigma_{ef} = \frac{\sigma_e}{\left(a\exp(\beta b)\right)^m} . \tag{23}$$

Fracture occurs when the condition $\sigma_{ef} = \sigma_F$ is satisfied.

5. Now it is worth noting some curious result which can be extracted from the data of experiments carried out by Bridgman [3] and Kolmogorov [4]. To find this, the curve of ultimate plastic strains (Fig.1) has to be reconstructed. By making use of expn (18) stress intensity can be excluded from the abscissa: now let the horisontal coordinate presents ratio σ_0/σ_F (Fig.2). Computations performed for three materials show that all the curves reveal explicit tendency to reach the magnitude $\sigma_0/\sigma_F = 1$ when $p_{eF}/p_F \to 0$. Thus, the characteristic of perfectly brittle strength which corresponds to equal tri-axial tension ($\sigma_0 = S_k$) can be determined: as it follows, it coincides with the true ultimate uni-axial tensile strength of the material ($S_k = \sigma_F$).

It should be noted that this result cannot be obtained using the adopted exponential approximation (16) because of some peculiarities of the function in vicinity of $p_e = 0$ (it seems that the latter has no phisical interpetation).

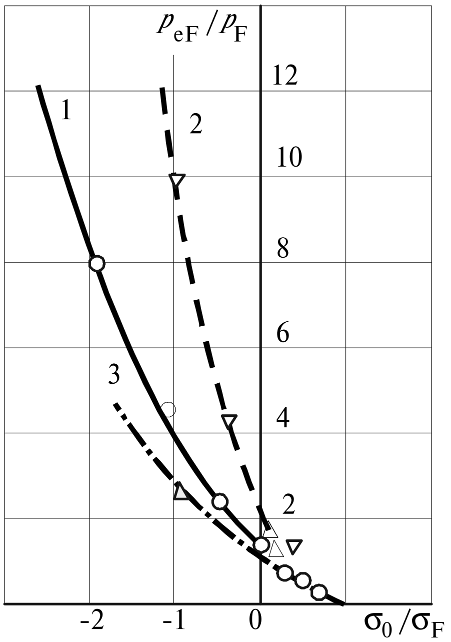

Fig. 2. On prediction of the pefectly brittle fracture characteristic
1 - steel; 2 - cast iron; 3 cupper.

6. Comparison of the deformation criterion and the above mentioned strength theories is illustrated by Fig.3. The values of parameters have been adopted (σ_F=1170MPa, τ_F=820MPa, m=0.067) correspond to titanium alloy. As it is seen, the disagreements between the theories are relatively large at biaxial compression and tension. Decrease of strength under biaxial equal tension and its increase under similar type of compression are distinctly reflected by Nadai's theory (line 2) and the suggested deformation criterion (line 3) while they are ignored by Mohr-Guest's (line 1) and Pysarenko-Lebedev's theories; it should be noted that the latter is constructed here in correspondence with the authors' recommendation.

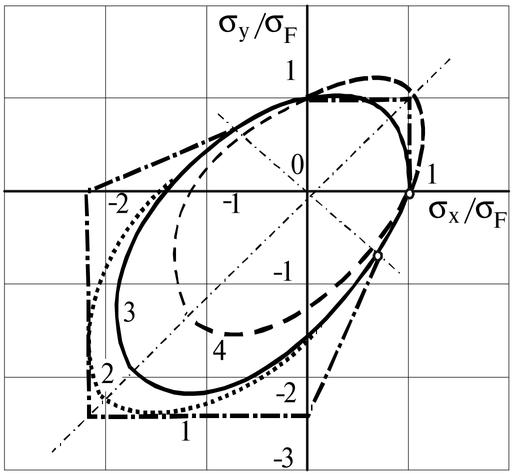

Fig. 3. Fracture criteria for plane stress state
— · — - Mohr - Guest; •••••• - Nadai;
——— - Deformation criterion;
— — — - Pisarenko-Lebedev.

Unfortunately, only a few experimental data are available to verify the correspondens of the strength theories. Konjushko [7] determined by tests under tension, compression and torsion the ultimate stresses for a group of thermally treated tool steels. The experimental values of ultimate strength (in MPa), of non-dimensional ratio $\sigma_F\sigma_F^c/\tau_F^2$ and parameter a by which the ultimate strain curve is determined are given in the Table .

Material	σ_F	σ_F^c	τ_F	$\sigma_F\sigma_F^c/\tau_F^2$	a
1.	19,8	41,0	17,2	2,74	1,5
2.	21,6	45,4	18,5	2,86	1,48
3.	21,3	51,0	18,3	3,21	1,49
4.	21,0	51,5	17,9	3,37	1,48
5.	15,8	31,4	13,8	2,6	1,51

It can be noted that the mentioned ratios prove to be rather close to the value predicted by the deformation criterion (accordingly to eqn (22) the latter value is equal to 3). Values of parameter a which were obtained testify that corresponding curves of ultimate strength for these steels practically coincide (Fig.1).

Fig.4 illustrates comparison between the fracture curves which correspond to theory suggested by Mohr and the deformation criterion against the data obtained by Coffin (see paper [6],reference [24]) for biaxial tension and tension-compression tests. For this aim the model based on the deformation criterion has been constructed (the tested material- grey cast iron):

$$\sigma_{eF}/\sigma_F = \left(3.62\exp(-3.86\beta)\right)^{0.4} \qquad (24)$$

($\sigma_F = 170\,MPa$, $\sigma_F^c = 480\,MPa$, $m = 0.4$). As it is seen, the latter criterion is characterized by somewhat better correspondence: the curve by which the theoretical prediction is presented proves to be almost over the all extent within spread of the experimental data. Meanwhile, the Mohr's theory has some deviations, especially in the first and third quadrants. Some possibilities of correction of the theory ("truncation" in the first quadrant and expansion in the third one) were discussed in [6].

7. Now let us consider the possibilities which have the deformation criterion for the long-term fracture prediction, i.e. under creep conditions. The following relation is valid for the rupture of viscous type caused by tension:

$$\dot{p}_{cm}t_F = p_{cF}; \qquad (25)$$

here $\dot{p}_{cm}$ is the mean creep rate; $t_F < 10^4$ -time to viscous fracture;

$$p_{cF} = \ln \frac{1}{1-\psi(t_F)} \cong \delta(t_F) \qquad (26)$$

where $\delta(t_F)$ is ordinary ultimate creep strain as a function of lifetime.

It can be assumed that

$$\dot{p}_{cm} = A(T)\sigma_{cF}^{n_1} \qquad (27)$$

($\sigma_{cF} = \sigma_{cF}(T, t_F)$ is long-term strength) where index $n_1 = an$, n -index of the Norton's rheological function:

$$\dot{p}_{\min} = A(T)\sigma^n \qquad (28)$$

(here isothermal loading conditions are supposed; $\dot{p}_{min}$ is the secondary creep rate).

For pearlitic and austenitic steels as well as for nickel alloys it can be adopted that $n_1/n = 1.08$ (coefficient of variation is equal 0,074) [8]. Thus, dependence of the long-term tensile strength on time t_F can be approximated by expression

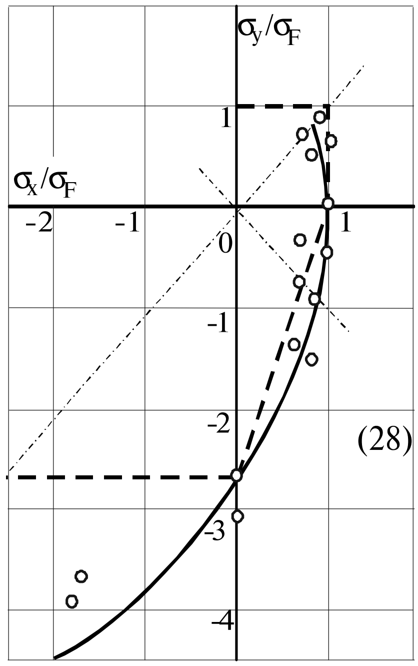

Fig. 4. Deformation (continuous line) and Mohr's criteria; comparison with experimental data obtainal by Coffin (see [6])

$$p_{cF} = A(T)\sigma_{cF}^{1.08n} t_F. \qquad (29)$$

As investigations show [5], the index of the rheological function does not depend on the type of combine loading. Thus, expn (28) may be generalized and takes the form:

$$\dot{p}_{e\,min} = A(T)s_e^n, \qquad (30)$$

where p_e is determined by expn (13) after its time differentiation, while s_e is defined by exps (5), (6). Accordingly, expression analogous to (29) in terms of the corresponding intensities (i.e. applicable at combine stresses) follows:

$$p_{ecF} = A(T)s_{eF}^{1.08n} t_F. \qquad (31)$$

Assuming the deformation criterion (16) remains valid and making use of exps (29), (31) we obtain an equation for any fixed lifetime

$$p_{ecF}/p_{cF} = (a\exp(\beta b))^{1/(1.08n)}, \quad \beta = \sigma_0/\sigma_e. \qquad (32)$$

Parameters a, b can be determined in a way similar to that in the case of short-term fracture (see expns (17)):here the corresponding experimental data obtained under creep conditions at some type of combined stresses (e.g. under torsion and biaxial tension of tube specimens subjected to internal pressure) are to be used.

In accordance with the data given in [8], the long-term strength curves obtained for stainless steel of the type 18-8 under tension and torsion at temperature 600°C are determined by equations:

$$\sigma_{cF} = 600 t_F^{-0.18}, \qquad \tau_{cF} = 444 t_F^{-0.18} \qquad (33)$$

$(\dim(\sigma, \tau) = MPa$, $\dim t_F = hours)$.

Hence, using exps (29), (31) the corresponding version of the deformation criterion has been constructed for the mentioned steel:

$$\sigma_{eF}/\sigma_F = \left(4.68\exp(-4.63\beta)\right)^{0.17}. \qquad (34)$$

Fig.5 illustrates comparison of the obtained limiting curve with those which follow from the empirical criteria suggested by Sdobyrev [9] and Trunin [10]; these criteria are widely used now in our country. Accordingly to the Sdobyrev's criterion the long-term fracture occurs if

$$\sigma_{ef} = \frac{1}{2}\left(\sigma_e + \frac{1}{2}\left(\sigma_1 + |\sigma_1|\right)\right) = \sigma_{cF}; \qquad (35)$$

analogous expression due to the criterion suggested by Trunin is:

$$\sigma_{ef} = mn^{1-3\sigma_0/m} = \sigma_{cF} \quad \text{where} \quad m = \frac{1}{2}\left(\sigma_e + \frac{1}{2}\left(\sigma_1 + |\sigma_1|\right)\right); n = \frac{2\sigma_{cF}}{\left(1+\sqrt{3}\right)\tau_{cF}}. \qquad (36)$$

For comparison, the corresponding expression for the effective stress which follows from the deformation criteria is-

$$\sigma_{ef} = \sigma_e\left(a\exp(\beta b)\right)^{-1/(1.08n)} \qquad (37)$$

the definitions given above for $\sigma_{cF} = \sigma_{cF}(t_F)$, $\tau_{cF} = \tau_{cF}(t_F)$ (see (33)) remain valid.

It should be noted that in the given expressions of the considered empirical long-term fracture criteria the initial (not true) stresses are meant. As it is seen from Fig.5, predictions obtained on the base of deformation criterion (continuous line) are rather close to those of the mentioned criteria which present some approximations of experimental data obtained by any of the authors. Only two types of tests were carried out: biaxial tension ($s_1/s_2 = 2$) and torsion. Perhaps, predictions based on the deformation criterion prove to be more adequate if the case of biaxial equal tension would be considered, but there is no data available to check. Unfortunately, we cannot comment here paper [11] while it is devoted to the subject.

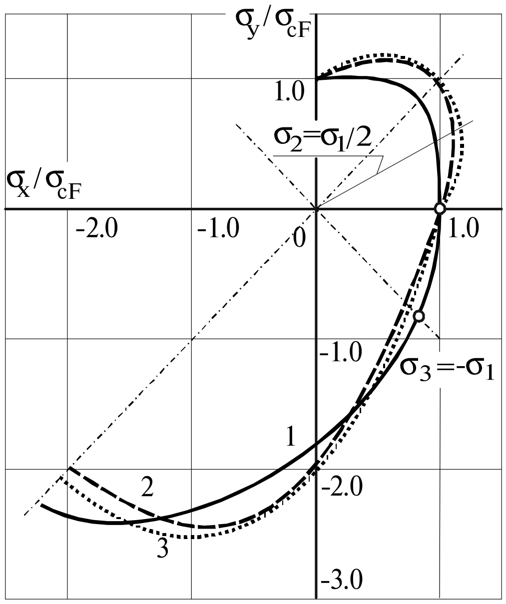

Fig. 5. Long-term fracture curve, comparizion of the suggested criteria (continuous line corresponds to the deformation one)

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